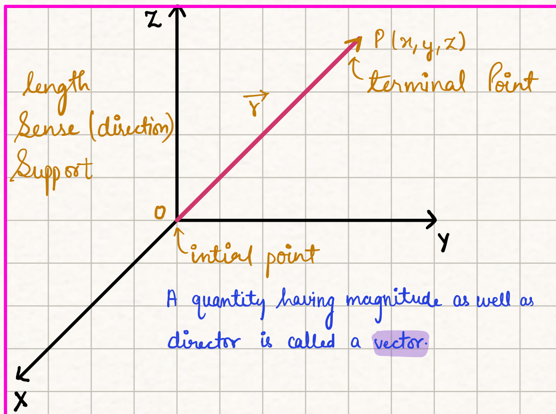


A quantity having magnitude as well as director is called a vector.

Position vector -

O (0,0,0) —— P (x,y,z)

$\vec{OP}$ where O and P as its intial and terminal points, is called Position vector of the point P with respect to O.

$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$

Types of Vector -

- **Zero Vector** - have zero magnitude or which has no fixed terminal points, it can go in any direction.
- **Equal Vectors** - same length, same sense or $\parallel^{al}$ or same direction.

$\vec{a}$ $\quad$ $\vec{b}$

$|\vec{a}| = k$ $\quad$ $|\vec{b}| = k$

$\vec{a}$

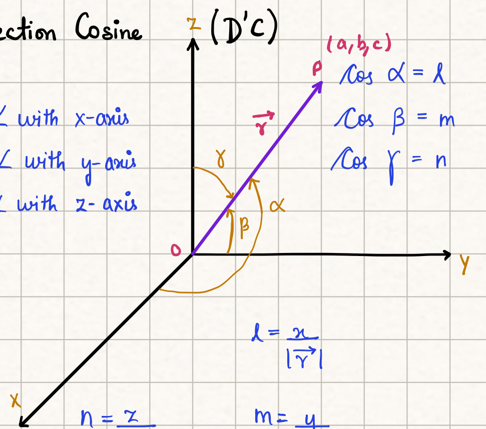
$\vec{b}$

- **like vector** - have same sense are called like vector.
- **Co-linear or parallel** - have same support, irrespective of length & sense
- **Co-Plannar** - Two vectors on same plane are co-plannar.
- **Co-intial** - same starting point

$\vec{OP} = a\hat{i} + b\hat{j} + c\hat{k}$

$D'C = \left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$

Direction Cosine



$\alpha \to \angle$ with x-axis

$\beta \to \angle$ with y-axis

$\gamma \to \angle$ with z-axis

$\cos\alpha = l$

$\cos\beta = m$

$\cos\gamma = n$

$\cos\alpha = \frac{a}{|\vec{AB}|}$

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$

$l = \frac{a}{|\vec{r}|}$

$|\vec{r}| = \frac{a}{l} = \frac{b}{m} = \frac{c}{n}$

$l = \frac{x}{|\vec{r}|}$

$n = \frac{z}{|\vec{r}|}$ $\qquad$ $m = \frac{y}{|\vec{r}|}$

$l^2 + m^2 + n^2 = 1$

$\frac{x^2}{|\vec{r}|^2} + \frac{y^2}{|\vec{r}|^2} + \frac{z^2}{|\vec{r}|} = 1$

$\frac{x^2+y^2+z^2}{|\vec{r}|^2} = 1 \Rightarrow \frac{\cancel{x^2+y^2+z^2}}{\{\cancel{\sqrt{x^2+y^2+z^2}}\}^2} = 1$

LHS = RHS

D'R (Direction Ratio)

where, components of $\vec{OP}$ are D'R.

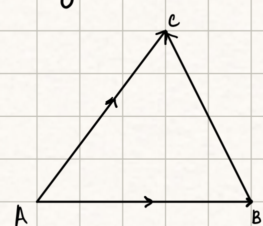
$D'R = \langle a, b, c \rangle$

Addition of Vectors

↳ Triangle law of Addition

↳ Parallelogram law of Addition

Triangle law

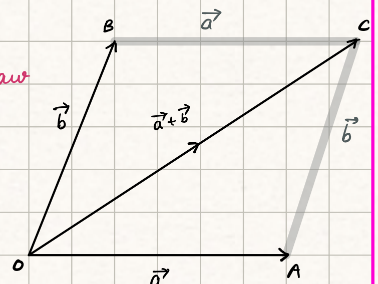


here,

$\vec{AC} = \vec{AB} + \vec{BC}$

Note: $\vec{BC'} = -\vec{BC}$

Parallelogram law



here,

$\vec{OA} + \vec{OB} = \vec{OC}$

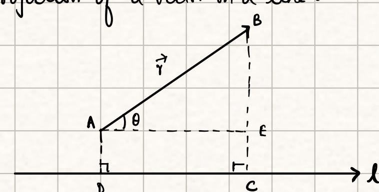
Properties of Vectors:

(1) Commutative property: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(2) Associative property: $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

Note: $|\lambda\vec{a}| = |\lambda||\vec{a}|$

Projection of a vector on a line:



In ΔABE -

$\cos\theta = \frac{|\vec{AE}|}{|\vec{r}|}$

$|\vec{AE}| = |\vec{r}| \cos\theta$

Projection of vector $\vec{r}$ on line $l = |\vec{r}| \cos\theta$

Projection of vector $\vec{a}$ on $\vec{b} = |\vec{a}| \cos\theta$

Projection of vector $\vec{b}$ on $\vec{a} = |\vec{b}| \cos\theta$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\vec{a} \cdot \vec{b} = |\vec{a}| \{\text{Projection of } \vec{b} \text{ on } \vec{a}\}$

$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$ = Projection of $\vec{b}$ on $\vec{a}$

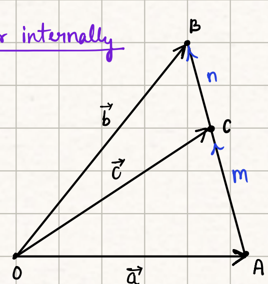
Projection of $\vec{b}$ on $\vec{a}$ = $\hat{a} \cdot \vec{b}$

Similarly, $\vec{a}$ projection on $\vec{b} = \vec{a} \cdot \hat{b}$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

Section formula

For internally



$\frac{\vec{AC}}{\vec{CB}} = \frac{m}{n}$

$\frac{\vec{OC} - \vec{OA}}{\vec{OB} - \vec{OC}} = \frac{m}{n}$

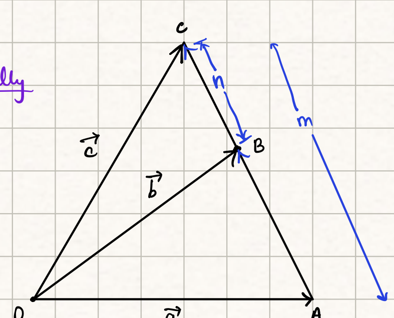
$\frac{\vec{c} - \vec{a}}{\vec{b} - \vec{c}} = \frac{m}{n}$

$n\vec{c} - n\vec{a} = m\vec{b} - m\vec{c}$

$\vec{c}(m+n) = m\vec{b} + n\vec{a}$

$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$

For externally



$\frac{\vec{AC}}{\vec{BC}} = \frac{m}{n}$

$\frac{\vec{OC} - \vec{OA}}{\vec{OC} - \vec{OB}} = \frac{m}{n}$

$\frac{\vec{c} - \vec{a}}{\vec{c} - \vec{b}} = \frac{m}{n}$

$n\vec{c} - n\vec{a} = m\vec{c} - m\vec{b}$

$\vec{c}(m-n) = m\vec{b} - n\vec{a}$

$\vec{c} = \frac{m\vec{b} - n\vec{a}}{m-n}$

Scalar Products of two vectors

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

θ = angle b/w $\vec{a}$ and $\vec{b}$

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Cross-Product of two vectors

$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta$

if, $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

So, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$